Katholieke Universiteit Leuven
Campus Kortrijk
Faculteit Wetenschappen
Departement Wiskunde

Workshop

Discrete Groups and Geometric Structures, with Applications.

(Crystallographic Groups and their Generalizations III)

at Kortrijk (Belgium), May 22 - 24, 2002.

Organized by

Paul Igodt, Karel Dekimpe, Wim Malfait (Kortrijk)
Herbert Abels (Bielefeld), Yves Félix (Louvain-la-Neuve), William Goldman (College Park), Fritz Grunewald (Düsseldorf), Kyung Bai Lee (Oklahoma)

Abstracts

It is a great nuisance that knowledge can only be acquired by hard work.

W. Somerset Maugham

Program

Wednesday, May 22

09.00-09.15 Welcome and Opening of the Workshop by P. Igodt.

09.15-10.15 B. Farb: *Group actions and Helly's Theorem.*

10.45-11.45 F. Labourie: *Surface groups and* $SL(n, \mathbb{R})$.

12.00-12.25

(LR1) V. Charette: *The Margulis invariant and the punctured torus.*

(LR2) M. Belliart: *On the dynamics of certain actions of free groups on closed real analytic manifolds.*

14.00-14.25

(LR1) I. Kim: *Affine action and Margulis invariant.*

(LR2) A. Descheemaeker: *Localizing infra-nilmanifolds.*

14.35-15.00

(LR1) K.-H. Jo: *Quasi-homogeneous convex domains and convex affine manifolds.*

(LR2) W. Malfait: *The structure of the (outer) automorphism group of a Bieberbach group.*

15.10-15.35

(LR1) J.-F. Quint: *Exponential divergence of discrete groups in higher rank.*

(LR2) L. Potyagailo: *Elementary splittings of Kleinian groups.*

15.40-16.20 Poster session

- L. Batten: *Planes of order* p^2 *with collineation groups of order* p^4.
- E. Breuillard: *On dense free subgroups of Lie groups.*
- I. Bumagin: *Conjugacy problem for relatively hyperbolic groups.*
- M. Molaei: *Top spaces.*
- A. Strojnowski: *Idempotents in group rings.*
- P. Witbooi: *Non-cancellation for groups with non-abelian torsion.*

16.25-16.50

(LR1) V. Bovdi: *Generalized crystallographic groups with indecomposable holonomy group.*

(LR2) B. Nucinkis: *Some examples of VF-groups.*

17.00-17.25

(LR1) K.B. Lee: *Maximal holonomy of infra-nilmanifolds with 2-dimensional quaternionic Heisenberg geometry.*

(LR2) V. Nekrashevych: *Iterated monodromy groups.*

Thursday, May 23

09.00-10.00 R. Grigorchuk: *Algorithmic problems and L-presentations of branch groups.*

10.15-11.15 D. Fried: *Affine flows on affine manifolds.*

11.25-11.50

(LR1) T. Barbot: *BTZ black holes and AdS geometry.*

(LR2) T. De Cat: *Translations in simply transitive affine actions of Heisenberg type Lie groups.*

12.00-12.25

(LR1) T. Kuessner: *Transversal fundamental groups and bounded cohomology.*

(LR2) O. Baues: *Realization problems for affine crystallographic groups.*

13.30 Conference picture with all participants

13.45 Departure to the hotels in Kortrijk

14.45 Departure to Ghent from Kortrijk-Center

15.30 Guided city tour to Ghent

17.30 Free shopping hour

18.30 Boat excursion on the Ghent waterways

19.30 Conference Dinner

Friday, May 24

09.00-10.00 Y. Benoist: *Projective deformations of hyperbolic structures.*

10.15-11.15 G. Soifer: *Affine crystallographic groups leaving a quadratic form of a signature $(n, 2)$ invariant.*

11.25-11.50

(LR1) J. Taback: *The large scale geometry of some metabelian groups.*

(LR2) A. Lichtman: *Restricted Lie algebras of soluble groups.*

12.00-12.25

(LR1) Y. Kamishima: *A quaternionic Carnot-Carathéodory structure on $(4n+3)$-manifolds and its geometric realization.*

(LR2) A. El Kacimi: *Currents invariant by a Kleinian group.*

14.00-14.25

(LR1) A. Akhmedov: *The girth of finitely generated groups.*

(LR2) V. Petrogradsky: *Algebras close to absolutely free algebras and enumeration of binary trees.*

14.35-15.00

(LR1) H. Pouseele: *The real cohomology of virtually nilpotent groups.*

(LR2) C. Frances: *Concerning the Ferrand-Obata theorem in Lorentzian geometry.*

15.30-15.55 K. Dekimpe: *Any polycyclic-by-finite group admits a NIL-affine crystallographic action.*

16.00-16.25 T. Drumm: *Signed Lorentzian displacement for Lorentzian transformations.*

16.30 Closing of the Workshop

Invited Talks

Projective deformations of hyperbolic structures.

Yves Benoist

Let M be a compact real hyperbolic manifold of dimension at least 3. By Mostow's rigidity theorem, this hyperbolic structure is unique. However, the underlying real projective structure may be continuously deformed. We will study the projective structures obtained by such deformations and we will show that for all of them the universal cover of M is, as a real projective variety, a strictly convex open set of the real projective space.

Group actions and Helly's Theorem.

Benson Farb

We describe a connection between the combinatorics of generators for certain groups and the combinatorics of Helly's 1913 theorem on convex sets. We use this connection to prove fixed point theorems for actions of these groups on nonpositively curved singular spaces, leading to a property we call FA_n, where FA_1 is Serre's property FA. The method applies to arithmetic and S-arithmetic groups in higher $\mathbb{Q}$-rank, to simplex reflection groups (including some non-arithmetic ones), and to higher rank Chevalley groups over polynomial and other rings (for example $\mathrm{SL}_n(\mathbb{Z}[x])$, $n > 2$).

Affine flows on affine manifolds.

David Fried

When the affine holonomy of an affinely flat manifold M preserves an affine vector field, one obtains a vector field on M. When M is compact, this vector field on M defines an affine flow. This gives some information about the developing image. In special cases, this process leads to constructions and classifications of certain types of affine manifolds. We will discuss several cases, including results of Smillie for diagonal holonomy and its application to constructing radiant flows without cross-sections. We also will discuss some three-dimensional results that use affine flows.

Algorithmic problems and L-presentations of branch groups.

Rostislav Grigorchuk

Branch groups are groups acting on rooted trees whose subnormal subgroup structure imitates the structure of the tree. They are extremely important for the study of just-infinite groups and many other topics of group theory and its applications. We shall discuss the word problem, the conjugacy problem and the generalized word problem for groups of branch type and groups generated by finite automata which often have a branch structure. Moreover, we shall consider the so called L-presentations of such groups, i.e. the presentations involving use of powers of an endomorphism of a free group over the generating set. This is related to the question about finite presentability of subgroups in direct products. Concrete examples, some general statements and some open problems will be given.

Surface groups and SL$(n, \mathbb{R})$.

François Labourie

In this talk, I will explain links between representations of surface groups (mainly in SL$(n, \mathbb{R})$), Anosov flows and curves in projective spaces generalizing the familiar picture known for $n = 2$, and the slightly less familiar picture for $n = 3$. I will in particular explain "geometric structures" of dynamic type underlying at least an open subset of these representations.

Affine crystallographic groups leaving a quadratic form of a signature $(n, 2)$ invariant.

Gregory Soifer

The well known Auslander conjecture says: every affine crystallographic group is virtually solvable (i.e. contains a solvable subgroup of finite index). This conjecture is still open. In our joint work with H. Abels, G. Margulis and myself, we proved this conjecture in the case when the crystallographic group leaves a quadratic form of a signature $(n, 2)$ invariant.

If the signature is $(n, 0)$, i.e. the form is positively defined and the corresponding affine group is just Euclidean, that fact is well known and due to Bieberbach (a so-called Bieberbach Theorem). If the signature is $(n, 1)$, the Auslander conjecture was proved by W. Goldman and Y. Kamishima. Our case, when the signature is $(n, 2)$, is principally different from the two previous and the proof needs new ideas and methods as well. The main goal of the talk is to present and explain them and also to inform the audience about some problems and conjectures in that direction.

Short Communications

Akhmedov, Azer: *The girth of finitely generated groups.*

The girth of a finitely generated group Γ is the supremum of the girthes of Cayley graphs (the length of minimal cycle in the graph) of Γ with the supremum being taken over all finite generating sets (see [1]). Thus, groups have finite or infinite girth type. Groups which satisfy a law (for example, finitely generated virtually solvable groups) have finite girth type (except $\mathbb{Z}$). But the class of groups with finite girth turns out to be much larger (although I do not know any finitely presented or residually finite example). Nevertheless, the following theorem is true.

> *Theorem:* Assume Γ is a noncyclic finitely generated linear (or word hyperbolic, or one-relator) group. Then Γ has infinite girth if and only if it is not virtually solvable.

I will discuss the main ideas of the proofs (see [2] for details).
It is an open question whether or not the girth type is a quasi-isometry invariant. I will briefly describe two potential counterexamples.
I will also introduce the so-called notion of Shreier girth of finitely generated groups and describe examples which shows that the analog of the theorem above is not true for none of the linear, word hyperbolic or one-relator groups.

[1] On the girth of groups. (Preprint)
[2] The girth of groups satisfying Tits Alternative. (Preprint)

Barbot, Thierry: *BTZ black holes and AdS geometry.*

In 1992, the physicists M. Banados, C. Teitelboim and J. Zanelli introduced a model of 3-dimensional spacetimes, the so-called BTZ black-holes, producing a typical behavior of "black hole": geometrically speaking, they are (noncompact) quotients of open subsets of the anti-de Sitter space, admitting a complete embedded spatial hypersurface, without periodic timelike or lightlike geodesics, and admitting non-complete light geodesics. These simple examples are then being developped to "multi-blackholes" and "wormholes" by several authors (D. Brill, S. Aminneborg, I. Bengtsson, ...)
In the mathematical point of view, these examples deserve some interesting geometric treatment. In some way, the task is to understand the domain where a discrete subgroup of AdS isometries does act properly and without periodic timelike geodesics: it is a problem analogous to the subject of the recent preprint [BCD] in the negative curvature case.

This talk will be an introduction to the topic, presenting what has be done on the subject, and addressing some questions. We will present a link with the work of G. Mess [Mess] on AdS geometry and the connection with the theory of earthquakes by W. Thurston.

[BTZ] M. Banados, M. Henneaux, C. Teitelboim, J. Zanelli, Geometry of the $2+1$ Black Hole, Phys. Rev. D (3) 48 (1993), 1506–1525.
[BCD] D. Brill, V. Charette, T. A. Drumm, Closed Timelike Curves in Flat Lorentz Spacetimes, math.DG/0201199
[Mess] G. Mess, Lorentz spacetimes of constant curvature, preprint IHES 90/28

Batten, Lynn M.: *Planes of order p^2 with collineation groups of order p^4.*

Planar functions were introduced by Dembowski and Ostrom in 1968 to describe projective planes possessing a collineation group with particular properties. It has since been shown that such projective planes must belong to one of three Lenz-Barlotti classes.
We establish the fact that any projective plane associated with a planar function defined over a finite field of order the square of a prime is necessarily Desarguesian. This is joint work with Robert Coulter and Marie Henderson.

Baues, Oliver: *Realization problems for affine crystallographic groups.*

Which virtually polycyclic groups can occur as fundamental groups of an affine space form? Some answers to this question may be obtained by considering the deformation theory of affine crystallographic groups.

Belliart, Michel: *On the dynamics of certain actions of free groups on closed real analytic manifolds.*

Let M be a closed connected real analytic manifold; let Γ be a free group on two generators. The set of analytic actions of Γ on M endowed with Taken's topology contains a nonempty open subset whose corresponding actions share three properties: (a) they have every orbit dense, (b) they leave invariant no geometric structure on M, (c) any homeomorphism conjugating two of them is analytic.

Bovdi, Viktor: *Generalized crystallographic groups with indecomposable holonomy group.*

Generalized crystallographic groups were introduced in [1]. According to the results of G. Hiss and A. Szczepánski [4] torsion free crystallographic groups with nontrivial irreducible holonomy group G do not exist. In [1,2] we studied the properties of generalized torsion-free crystallographic groups with indecomposable holonomy groups isomorphic to either a cyclic group or a direct product of two cyclic groups of order p. In our talk, using the theory of integral representations of finite groups (see, for example, [5]), we discuss the properties of generalized crystallographic groups with indecomposable holonomy group which is isomorphic to either a direct product of two cyclic groups of order p or the cyclic group of order C_{p^s}.
In particular, up to isomorphism, we give the full description of generalized crystallographic groups with an indecomposable holonomy group, which is isomorphic to the direct product of two cyclic groups of order 2.

[1] Bovdi, V.A.; Gudivok, P.M.; Rudko, V.P., Torsion free groups with indecomposable holonomy group I, J. Group Theory 5, 2002, 75–96
[2] -, Torsion free groups with indecomposable holonomy group II, To appear, 2002.
[3] Bovdi, V.A., Generalized crystallographic groups with the indecomposable holonomy group, Preprint, 2002.
[4] Hiss, G.; Szczepánski, A., On torsion free crystallographic groups, J. Pure and Appl. Algebra, 1(74), 1991, 39–56.
[5] Plesken, W., Some applications of representation theory, Prog. in Math., 95, 1991, 477-496

Breuillard, Emmanuel: *On dense free subgroups of Lie groups.*

We investigate dense free subgroups of connected real Lie groups. In particular we show that any dense subgroup of a connected semisimple real Lie group contains a dense subgroup, which is free on two generators. More generally, if G is any non solvable connected real Lie group, then any finitely generated dense subgroup of G contains a dense free subgroup of rank $\leq 2\dim(G)$.

Bumagin, Inna: *Conjugacy problem for relatively hyperbolic groups.*

Solvability of the conjugacy problem for relatively hyperbolic groups was announced by Gromov. We prove this assertion, using the definition of Farb of a relatively hyperbolic group. We conclude that the conjugacy problem is solvable for fundamental groups of complete, finite-volume, negatively curved manifolds, and also for fully residually free groups.

Charette, Virginie: *The Margulis invariant and the punctured torus.*

Suppose γ is a freely acting affine transformation of $\mathbb{R}^{2,1}$ whose linear part is hyperbolic. The Margulis invariant measures signed Lorentzian length on the unique γ-invariant geodesic. In order for an affine deformation of a Schottky group Γ to act freely and properly discontinuously on $\mathbb{R}^{2,1}$, the sign of the Margulis invariant must remain constant; whether the converse holds remains unknown. We will discuss the case when the linear part of Γ is the fundamental group of the punctured torus.

De Cat, Tine: *Translations in simply transitive affine actions of Heisenberg type Lie groups.*

(Joint work with Karel Dekimpe and Paul Igodt)
Let G be a 2-step nilpotent Lie group with a 1-dimensional commutator subgroup. We prove that for any simply transitive and affine action of G, there exists a nontrivial subgroup of G acting as pure translations. This result no longer holds in case the commutator subgroup is higher dimensional.

Dekimpe, Karel: *Any polycyclic-by-finite group admits a NIL-affine crystallographic action.*

We prove that any polycyclic-by-finite group admits a properly discontinuous and cocompact action on a simply connected, connected nilpotent Lie group N. This action is affine in the sense that the group acts as a subgroup of the affine group $\mathrm{Aff}(N) = N \rtimes \mathrm{Aut}(N)$ of connection preserving automorphisms of N. As a consequence, we obtain that any polycyclic-by-finite group admits a polynomial crystallographic action of degree bounded above by its Hirsch length.

Descheemaeker, An: *Localizing infra-nilmanifolds.*

Motivated by previous results of various authors on the P-localization (with P any set of primes) of $K(G,1)$-spaces with G a finite or a nilpotent group, we study the effect of P-localization on infra-nilmanifolds.
We show that the P-localization of an infra-nilmanifold is a virtually nilpotent space which is either aspherical or has infinitely many nonzero homotopy groups. Moreover, we develop criteria to decide for a given infra-nilmanifold to which of those two homotopy types its P-localization belongs. In fact, we prove that the P-localization of an orientable infra-nilmanifold is aspherical if and only if its holonomy group is P-torsion. The same holds for non-orientable infra-nilmanifolds if 2 is in P. Otherwise, the situation is more complicated, as we will illustrate.

Drumm, Todd: *Signed Lorentzian displacement for Lorentzian transformations.*

In a remarkable paper on the existence of complete locally flat affine manifolds with free fundamental group, Margulis defined the *signed Lorentzian length* of a closed geodesic associated to a hyperbolic Lorentzian translation and exploited its properties to great effect. The author and V. Charette have since extended this notion to a *signed Lorentzian displacement* for hyperbolic and parabolic Lorentzian transformations (though there are no corresponding closed geodesics for parabolic transformations). We will discuss the signed Lorentzian displacement and its application in several cases. In particular, we will comment on the author and W. Goldman's isospectrality result for complete Lorentzian 3-manifolds and Guediri's results on the nonexistence of closed timelike geodesics on certain compact Lorentzian manifolds.

El Kacimi, Aziz : *Currents invariant by a Kleinian group.*

(joint work with S. Matsumoto and T. Moussa)
Let B^{n+1} be the unit ball of $\mathbb{R}^{n+1}$ equipped with the hyperbolic metric. The group $\mathrm{Iso}_+(B^{n+1})$ of orientation preserving isometries of B^{n+1} is identified with the group $\mathrm{Conf}_+(S^n)$ of orientation preserving conformal transformations of the sphere S^n (viewed as the boundary of B^{n+1}) with its canonical metric. A discrete subgroup Γ of $\mathrm{Conf}_+(S^n)$ is called a *Kleinian group*. Let Λ be the *limit set* of Γ, $\Omega = S^n \setminus \Lambda$ its domain of discontinuity and δ its *critical exponent*. The sets Λ and Ω are Γ-invariant and the action of Γ on Ω is free and proper. Let $\mathcal{C}_p^\Gamma(S^n)$ and $\mathcal{C}_p^\Gamma(\Omega)$ denote the spaces of Γ-invariant currents respectively on S^n and Ω; let $\mathcal{C}_p^\Gamma(S^n,\Lambda)$ be the space of Γ-invariant currents on S^n whose support is in Λ. We have an exact sequence $0 \longrightarrow \mathcal{C}_p^\Gamma(S^n,\Lambda) \hookrightarrow \mathcal{C}_p^\Gamma(S^n) \xrightarrow{L_p} \mathcal{C}_p^\Gamma(\Omega)$ where L_p is the *localization map* (*i.e.* L_p

restricts a p-current on S^n, to a p-current on Ω); L_p is not surjective in general. We have the following

> *Theorem:* Suppose that the quotient manifold Ω/Γ is compact and that $p > \delta$. Then the localization map L_p is sujective.

Frances, Charles: *Concerning the Ferrand-Obata theorem in Lorentzian geometry.*

Given a Riemannian conformal structure $\mathcal{C}$ on a compact manifold M, we look at the conformal group of $(M, \mathcal{C})$. For which $(M, \mathcal{C})$ is this group non-compact? A striking result of Ferrand and Obata asserts that the only case for which such a phenomenon occurs is the case of the sphere S^n endowed with its canonical conformal structure. In other words, (S^n, can) is the only compact conformal manifold whose conformal group is not reducible to some isometry group, up to conformal change of metric. Such a manifold is called *essential*.
The aim of the talk is to study the generalization of Ferrand-Obata theorem to Lorentzian context. Can we expect a list of compact Lorentzian manifolds with an essential conformal group?
We will introduce Einstein's space $\mathbb{EIN}_n$, which is the conformal lorentzian analogue of the sphere. Making quotients of open domains of $\mathbb{EIN}_n$ by suitable Schottky groups, we'll give examples of construction of essential Lorentzian compact manifolds.

Jo, Kyeong-hee: *Quasi-homogeneous convex domains and convex affine manifolds.*

A convex affine n-manifold is a quotient of a convex affine domain Ω in $\mathbb{R}^n$ by a discrete subgroup Γ of $\mathrm{Aff}(n, \mathbb{R})$ acting on Ω properly discontinuously and freely. So the study of compact convex affine manifolds is equivalent to the study of divisible convex affine domains and their automorphisms.
We study which convex affine domain can cover a compact affine manifold for dimension ≤ 4. For this purpose, we first show that for each n a paraboloid is the only strictly convex quasi-homogeneous affine domain in $\mathbb{R}^n$ up to affine equivalence and then find out all possible types of shapes for developing images of compact convex affine manifolds with dimension ≤ 4.
As an application, we show that Markus conjecture is true for convex affine manifolds with dimension ≤ 4, that is, any compact convex affine manifold with dimension ≤ 4 is complete if it has parallel volume.

Kamishima, Yoshinobu: *A quaternionic Carnot-Carathéodory structure on $(4n+3)$-manifolds and its geometric realization.*

We consider an integrable, nondegenerate, quaternionic Carnot-Carathéodory structure on a $(4n+3)$ - manifold M. More precisely, a quaternionic Carnot-Carathéodory structure on a $(4n+3)$-manifold M is given by an exact sequence $1 \to B \to TM \xrightarrow{\theta} L \to 1$ where L is a 3-dimensional vector bundle with certain gluing condition, and θ (viewed as a L-valued 1-form) satisfies that

$$\theta \wedge \theta \wedge \theta \wedge \overbrace{d\theta \wedge \cdots \wedge d\theta}^{2n-\text{times}} \neq 0 \text{ in } \mathbf{R} \subset \mathbf{H} = \Gamma(M, \Omega^{4n+3}(L)).$$

In order to study the geometric structure, we assign to B an $\mathfrak{sp}(1)$-valued 1- form ω globally defined on M.
Letting $\omega = \omega_1 \boldsymbol{i} + \omega_2 \boldsymbol{j} + \omega_3 \boldsymbol{k}$ where each ω_a is a real valued 1-form and $\{J_a\}_{a=1,2,3}$ a quaternionic structure on B, we examine topology and geometry of such compact $(4n+3)$- manifolds $(M, (\omega_1, \omega_1, \omega_3), (J_1, J_2, J_3))$. Especially, we classify compact manifolds when the quaternionic Carnot-Carathéodory curvature tensor vanishes. They are uniformized with respect to the geometry $(\mathrm{Psp}(n+1,1), S^{4n+3})$.

Kim, Inkang: *Affine action and Margulis invariant.*

In this paper we show that two Zariski dense subgroups consisting of hyperbolic elements in $SO(n+1,n) \ltimes \mathbb{R}^{2n+1}$ with the same marked Margulis invariant, are conjugate. We also consider in affine deformations an analogue of quasifuchsian deformation of Fuchsian groups.
The theorems we want to prove are

> *Theorem A:* Let Γ_1 and Γ_2 be Zariski dense subgroups of $SO(n+1,n) \ltimes \mathbb{R}^{2n+1}$ consisting of hyperbolic elements. Suppose $\phi : \Gamma_1 \to \Gamma_2$ is an isomorphism preserving the Margulis invariant. Then Γ_1 and Γ_2 are conjugate. Specially if ϕ is such that $\phi(A,b) = (A,c)$, i.e., Γ_1 and Γ_2 have the identical linear parts, then ϕ is a conjugation by a translation.
>
> *Theorem B:* Suppose $\Gamma \subset SO(2,1)$ is a cocompact lattice. Then any affine deformation $\phi : \Gamma \to SO(2,1) \ltimes \mathbb{R}^4$ with u_ϕ positive is not proper.

Kuessner, Thilo: *Transversal fundamental groups and bounded cohomology.*

We consider the foliated Gromov norm $\parallel M \parallel_{\mathcal{F}}$, defined by Calegari, which is a refinement of Gromov's simplicial volume $\parallel M \parallel$ to the case of foliated manifolds $(M, \mathcal{F})$.
It was known since Gromov's work that properties of the simplicial volume are strongly related to the fundamental group. In our talk, we define a notion of transversal fundamental group $\pi_1^{\mathcal{F}} M$ for foliated manifolds and extend some of Gromov's results to the foliated case. For example, we prove: if $\pi_1^{\mathcal{F}} M = 0$, then $\parallel M \parallel_{\mathcal{F}} = 0$.
One has a canonical surjection $\pi_1^{\mathcal{F}} M \to \pi_1 M$. Injectivity of this map is a sufficient (but probably not necessary) condition for the equality $\parallel M \parallel_{\mathcal{F}} = \parallel M \parallel$. (This generalises Calegari's theorem that $\parallel M \parallel_{\mathcal{F}} = \parallel M \parallel$ if $\mathcal{F}$ branches in at most one direction, e.g., is a fibration.)
We study an analogous notion of transversal Gromov norm for contact structures.

Lee, Kyung Bai and Ha, Ku Young, and Lee, Jong Bum: *Maximal holonomy of infra-nilmanifolds with 2-dimensional quaternionic Heisenberg geometry.*

Let $\mathbf{Q}_{4n+3}$ be the quaternionic Heisenberg group of real dimension $4n + 3$ and I_{n+1} denote the maximal order of the holonomy groups of all infra-nilmanifolds with $\mathbf{Q}_{4n+3}$-geometry. We prove that $I_2 = 48$. As an application, by applying Kim and Parker's result, we obtain the minimum volume of a 2-dimensional quaternionic hyperbolic orbifold with k cusps is at least $\frac{\sqrt{2}k}{720}$.

Lichtman, Alexander: *Restricted Lie algebras of soluble groups.*

Let H be a group. We recall that a sequence of normal subgroups

$$H = H_1 \supseteq H_2 \supseteq \dots$$

is a p-series in H if $[H_i, H_j] \subseteq H_{i+j}$ and the inclusion $x \in H_i$ implies that $x^p \in H_{pi}$.
If a p-series is given then a restricted Lie algebra associated to this p-series can be constructed by the classical article of Lazard [1]. We denote this restricted Lie algebra by $L_p(H, H_i)$ and its universal p-envelope by $U_p(L_p(H, H_i))$. Our first main result is the following theorem.

Theorem 1: Let H be a poly-{infinite cyclic group}. Assume that H contains no elements of infinite p-height. Then there exists in H a p-series such that the restricted Lie algebra $L_p(H, H_i)$ associated to this p-series is free abelian.

The series H_i $(i = 1, 2, \ldots)$ in Theorem 1 differs from the Lazard-Zassenhaus series of dimension subgroups and we construct an example that shows that the p-envelope of the restricted Lie algebra corresponding to the Lazard-Zassenhaus series may contain nilpotent elements.
Our second result is *Theorem* 2. Let F be a free group with a free system of generators x_i $(i \in I)$, N be a normal subgroup of F. Let $V(N)$ be a verbal subgroup of N such that the quotient group $\bar{N} = N/V(N)$ is torsion free nilpotent, and let $\bar{F} = F/V(N)$. We obtain in Theorem 2 a description of the restricted Lie algebra $L_p(\bar{F})$ associated to the Lazard-Zassenhaus series of dimension subgroups of $\bar{F}$. A special example of the groups $\bar{F} = F/V(N)$ are groups which are free in a product of two varieties. When the quotient group F/N is finite and $V(N) = N'$ the group $\bar{F}$ is a crystallographic group.
We apply Theorems 1 and 2 together with the methods of [2] in order to construct discrete valuations in the group algebras over a field K of characteristic p. We prove that there exists a valuation ρ in the group algebra KH or $K\bar{F}$ such that the graded ring associated to this valuation is isomorphic to a polynomial ring in a finite or countable number of variables. We apply this valuation for study of the skew field of fractions of these algebras.
We make an essential use of results and methods from [3].

[1] M. Lazard, Sur les groupes nilpotents et les anneaux de Lie, Annales Scientifique de l'ecole normale Superieure (3) (1954), 101-190.
[2] A. I. Lichtman, Matrix rings and linear groups over a field of fractions of enveloping algebras and group rings, I, J. Algebra 88 (1984), 1-37.
[3] A.I. Lichtman, Valuation methods in group rings and in skew fields, I (to appear), J. Algebra .

Malfait, Wim and Szczepanski, Andrzej: *The structure of the (outer) automorphism group of a Bieberbach group.*

Let M be a compact flat Riemannian manifold of dimension n and assume that $E = \pi_1(M)$ denotes its fundamental group. Then E is a torsion–free group fitting into an extension $\mathbb{Z}^n \rightarrowtail E \twoheadrightarrow F$ where $\mathbb{Z}^n$ is maximal abelian in E and F is a finite group. We refer to F as the holonomy group of $M = E\backslash\mathbb{R}^n$ and E is said to be a Bieberbach group. The holonomy group F acts on $\mathbb{Z}^n$ by conjugation in E, defining a faithful representation $T : F \to \mathrm{Gl}(n, \mathbb{Z})$. It is known that $\mathrm{Out}(E)$ is finite if and only if all $\mathbb{Q}$-irreducible components in the $\mathbb{Q}$-decomposition of the holonomy representation T are $\mathbb{R}$-irreducible and of multiplicity one. Essentially, the proof reduces to analyzing when the normalizer of $T(F)$ in $\mathrm{Gl}(n, \mathbb{Z})$ is finite.
Motivated by Tits' alternative, we develop a necessary and sufficient condition to decide whether the normalizer of a finite group of integral matrices is polycyclic-by-finite or is containing a non-abelian free group. If $T : F \to \mathrm{Gl}(n, \mathbb{Z})$ is a faithful representation of a finite group F, then the normalizer of $T(F)$ in $\mathrm{Gl}(n, \mathbb{Z})$ is polycyclic-by-finite if and only if all components in the $\mathbb{Q}$-decomposition of T occur with multiplicity one

and the Schur index over $\mathbb{Q}$ of each $\mathbb{Q}$-irreducible component which is $\mathbb{R}$-reducible is equal to one. This result translates in a straightformard way to a criterion for the (outer) automorphism group of a Bieberbach group to be polycyclic-by-finite (or alternatively, to contain a non-abelian free group).

Molaei, Mohammad Reza: *Top spaces.*

We will introduce Top spaces. Top spaces are a generalization of Lie-groups, and they will be introduced by the use of Clifford semi-groups. We will show that one can make use of these spaces to construct new Lie-algebras.

Nekrashevych, Volodymyr: *Iterated monodromy groups.*

We define a class of groups associated with branched coverings (for instance by rational mappings of the complex sphere). These group are acting on regular rooted trees in a self-similar way, are often generated by finite automata and have many exotic properties. We will show how the Julia set of an expanding mapping (in particular, of a postcritically finite rational function) can be reconstructed from the iterated monodromy group.

Nucinkis, Brita: *Some examples of VF-groups.*

The proposed talk describes joint work with Ian J. Leary.
Let G be a discrete group. A model for an $\underline{E}G$ is a $G-CW$-complex X, such that all cell stabilizers are finite and such that X^K is contractible for each finite subgroup K. A group H is said to be of type F is there is a finite model for EH and a group G is of type VF is it contains a finite index subgroup of type F. We construct examples of VF-groups which do not have finite-type model for $\underline{E}G$.
A theorem of F.X.Connolly and T. Kożniewski (for groups of finite vcd) and W.Lück (in general) states that there is a finite type $\underline{E}G$ if and only if
(a) G contains only finitely many conjugacy classes of finite subgroups;
(b) for each finite subgroup $P \leq G$, the normalizer $N = N_G(P)$ is of type F_∞.

Our groups G, for which either (a) or (b) fails are constructed as follows. Let L be a finite flag-complex and G_L be the right-angled Artin group, which has generators the vertices of L subject to the relations that the ends of every edge commute. Denote by H_L the kernel of the canonical homomorphism $G_L \to \mathbb{Z}$, which sends every generator to 1. M. Bestvia and N. Brady showed that H_L is of type F if and only L is contractible. Let Q be a finite group of Automorphisms of L. Our groups G are then constructed as semidirect products $G = H_L \rtimes Q$, which are of type VF if and only if the finite flag complex L is contractible (by the theorem of Bestvina-Brady). The case where G contains infinitely many conjugacy classes of finite subgroups (i.e. (a) fails for G) corresponds to the case when the fixed-point set L^Q is empty. The case when (b) fails for the normalizer $N_G(Q)$ corresponds to the case when L^Q is non-empty but is not contractible.
It turns out that each of our groups embeds in $GL_m(\mathbb{Z})$ for sufficiently large m. For example we can give an explicit presentation of a group G of type VF that contains infinitely many conjugacy-classes of subgroups isomorphic to $A_5 \times C_2$ and which can be embedded in $SL_{46}(\mathbb{Z})$.

Petrogradsky, Victor: *Algebras close to absolutely free algebras and enumeration of binary trees.*

We study three classes of algebras: absolutely free algebras, free nonassociative commutative, and free nonassociative noncommutative algebras. We study asymptotics of growth for free algebras of these varieties as well as for their subvarieties. Mainly, we study finitely generated algebras. The following subvarieties are studied in these classes: nil-algebras, solvable algebras and completely solvable algebras.
These results are equivalent to an enumeration of binary labelled and unlabelled rooted trees that does not contain some forbidden subtrees. We enumerate these trees using generating functions. In case of nil-algebras and completely solvable algebras the respective functions are rational. For solvable algebras the generating functions are algebraic.
It is known that these three varieties of algebras satisfy Schreier's property, i.e. any subalgebras of free algebras are free. For free groups, there is Schreier's formula for the rank of a subgroup of the free group. We find analogues of this formula for these varieties. They are written in terms of series. In particular, we study invariants of finite groups acting on absolutely free algebras.

Potyagailo, Leonid: *Elementary splittings of Kleinian groups.*

This is a joint paper with T. Delzant. A Kleinian group is a discrete subgroup of the hyperbolic space $\mathbb{H}^n$. We study graph of groups decompositions with elementary (i.e. virtually abelian) edge groups of a wide class of geometrically finite Kleinian groups. As an application of our method we obtain a criterion for such a group to be cohopfian (i.e. not to contain a proper subgroup isomorphic to itself). Some related results (e.g. relative cohopficity and different types of the accessibility for such splittings)) are also discussed.

Pouseele, Hannes: *The real cohomology of virtually nilpotent groups.*

When a torsion free finitely generated nilpotent group G acts properly discontinuously and cocompactly on a contractible differentiable manifold X, we know that the real group cohomology of G is isomorphic to the deRham cohomology of the quotient space $G\backslash X$. We show that in case the group G acts via polynomial maps on some space $\mathbb{R}^n$, this deRham cohomology is in fact the cohomology of a complex of G-invariant polynomial differential forms on $\mathbb{R}^n$. This generalizes a theorem of D.Fried, W.Goldman and M.W.Hirsch, who obtained this result for affine actions.
As any finitely generated torsion free nilpotent group admits such a polynomial action (which is easy to construct), this approach to computing cohomology is possible for all such groups. It turns out that the computation of the cohomology of the complex of G-invariant polynomial differential forms is in fact a trivial linear algebra problem. Moreover, these results can be generalized to the case of all virtually nilpotent groups, for which we also obtain a kind of Poincaré duality result.
As an application, we present explicit formulas for the Betti numbers of any finitely generated virtually abelian group.

Quint, Jean-François: *Exponential divergence of discrete groups in higher rank.*

Let G be a semisimple group and let Γ be a discrete Zariski dense subgroup of G. We introduce a homogeneous function, defined on a Weyl chamber of G, which generalizes in higher rank the so-called exponent of convergence usually considered in $\mathbb{R}$-rank one. We show that this function is concave, from what we deduce generalizations of the Patterson-Sullivan construction of conformal densities.

Strojnowski, Andrzej : *Idempotents in group rings.*

Let R be a ring not necessary commutative. We assign to each R a class of groups, so called $E(R)$ groups. If G is an $E(R)$ group then the group ring RG contains nontrivial idempotents if and only if R contains nontrivial idempotents. The class $R(G)$ does almost not depend on the ring R, contains torsion-free solvable groups and is closed under some groups operations as extension, direct and free products.

Taback, Jennifer: *The large scale geometry of some metabelian groups.*

(Joint with Kevin Whyte)
We study the large scale geometry of the upper triangular subgroup of $PSL_2(\mathbb{Z}[1/n])$, which arises naturally in a geometric context as the cusp subgroup. We classify these groups up to quasi-isometry and show that they are quasi-isometrically rigid with infinite dimensional quasi-isometry group. We generalize our results to a larger class of groups which are metabelian and are higher dimensional analogues of the solvable Baumslag-Solitar groups $BS(1,n)$.
All of these groups have a beautiful geometry reliant on their many Baumslag-Solitar subgroups. One way to understand these groups is by analyzing these subgroups and finding a "horobrick" building block to construct a complex on which they act. I will describe these groups geometrically as well as more algebraically.

Witbooi, Peter: *Non-cancellation for groups with non-abelian torsion.*

For any group G, by $\chi(G)$ we denote the set of all isomorphism classes of groups H such that $H \times \mathbb{Z} \simeq G \times \mathbb{Z}$. If G is finitely generated with finite commutator subgroup, then $\chi(G)$ has a group structure. We develop methods for computing the groups $\chi(G)$ and perform such computations. For a finite normal subgroup F of G, we compare $\chi(G)$ with $\chi(G/F)$.

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